CBSE 2026 EXAMINATIONS
XII MATHEMATICS (041)
Series RPSQ4 ♦ Q.P. Code 65/4/1; 65/4/2; 65/4/3

Time Allowed : 180 Minutes **Max. Marks : 80**

General Instructions :

1. This Question paper contains **five sections - A, B, C, D and E**. Each section is compulsory. However, there are **internal choices** in some questions.
2. Section A has **18 MCQs** and **02 Assertion-Reason (A-R)** based questions of **1 mark** each.
   Section B has **05 questions** of **2 marks** each.
   Section C has **06 questions** of **3 marks** each.
   Section D has **04 questions** of **5 marks** each.
   Section E has **03 Case-study / Source-based / Passage-based** questions with **sub-parts** (**4 marks** each).
3. There is no overall choice. However, **internal choice** has been provided in
   - **02 Questions of Section B**
   - **03 Questions of Section C**
   - **02 Questions of Section D**
   - **02 Questions of Section E**

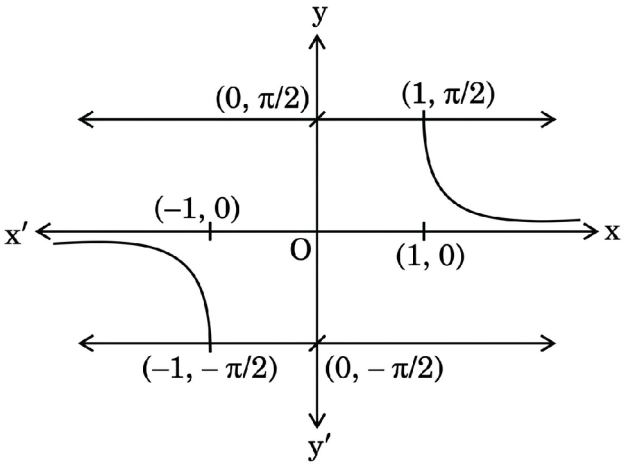
   You have to attempt only one of the alternatives in all such questions.

☑ Q.P. Code - 65/4/1

SECTION A

This section has 20 multiple choice questions of 1 mark each.

01. The following graph represents



(a) $y = \cos^{-1} x$ (b) $y = \sec^{-1} x$ (c) $y = \tan^{-1} x$ (d) $y = \text{cosec}^{-1} x$

Sol. (d) Since for $y = \text{cosec}^{-1} x$, $x \in (-\infty, -1] \cup [1, \infty)$ and $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right] - \{0\}$.

02. If $A = \left[a_{ij}\right]$ is a 2×2 matrix whose elements are given by $a_{ij} = \frac{|i - 3j|}{2}$, then A' is

(a) $\begin{bmatrix} 1 & \frac{5}{2} \\ \frac{1}{2} & 2 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & \frac{1}{2} \\ \frac{5}{2} & 2 \end{bmatrix}$ (c) $\begin{bmatrix} 2 & \frac{5}{2} \\ \frac{1}{2} & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 2 & \frac{1}{2} \\ \frac{5}{2} & 1 \end{bmatrix}$

Sol. (b) Let $A=\left[a_{ij}\right]_{2\times 2}=\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$

$\Rightarrow A=\begin{bmatrix} 1 & \frac{5}{2} \\ \frac{1}{2} & 2 \end{bmatrix} \qquad \therefore A'=\begin{bmatrix} 1 & \frac{1}{2} \\ \frac{5}{2} & 2 \end{bmatrix}$

03. The principal value of $\sec^{-1}(\sqrt{2})+2\,\text{cosec}^{-1}(-\sqrt{2})$ is

(a) $-\frac{\pi}{2}$ (b) $-\frac{\pi}{4}$ (c) $\frac{\pi}{4}$ (d) $\frac{\pi}{2}$

Sol. (b) $\sec^{-1}(\sqrt{2})+2\,\text{cosec}^{-1}(-\sqrt{2})=\frac{\pi}{4}+2\left(-\frac{\pi}{4}\right)=\frac{\pi}{4}-\frac{\pi}{2}=-\frac{\pi}{4}$.

04. If points (2, 3), (0, 4) and (p, 2) are collinear, then the value of p is

(a) $\frac{4}{7}$ (b) $-\frac{3}{7}$ (c) 4 (d) –4

Sol. (c) Since points are collinear so, $\begin{vmatrix} 2 & 3 & 1 \\ 0 & 4 & 1 \\ p & 2 & 1 \end{vmatrix}=0$

$\Rightarrow 2(4-2)-3(0-p)+1(0-4p)=0$

$\Rightarrow 4+3p-4p=0$

$\therefore p=4$.

05. Differential of e^{e^x} with respect to x is

(a) $\log x$ (b) e^{e^x} (c) $e^x e^{e^x}$ (d) $(e^x)^2$

Sol. (c) $\frac{d}{dx}\left[e^{e^x}\right]=e^{e^x}\times e^x$.

06. The surface area of a sphere when its volume changes at the same rate as its radius is

(a) 4π sq. units (b) 1 sq. unit (c) 4 sq. units (d) π sq. units

Sol. (b) Since $\frac{dV}{dt}=\frac{dr}{dt}$

$\Rightarrow \frac{d}{dt}\left[\frac{4\pi}{3}r^3\right]=\frac{dr}{dt}$

$\Rightarrow 4\pi r^2\times\frac{dr}{dt}=\frac{dr}{dt}$

$\Rightarrow 4\pi r^2=1$

$\therefore$ Surface area of sphere $=1$ sq. unit.

07. If $f(x)=\begin{cases} \frac{\sin x}{x}+\cos x, & x\neq 0 \\ k, & x=0 \end{cases}$ is a continuous function at $x=0$, then the value of k is

(a) 0 (b) –2 (c) –1 (d) 2

Sol. (d) As f is continuous at $x=0$ so, $f(0)=\lim\limits_{x\to 0} f(x)$

$\Rightarrow k=\lim\limits_{x\to 0}\left(\frac{\sin x}{x}+\cos x\right)$

$\Rightarrow k=1+\cos 0=1+1$

$\therefore k = 2$.

08. The greatest integer function, $f(x) = [x],\ 0 < x < 3$ is **not** differentiable at how many points?

(a) At only one point (b) At only two points
(c) At no point (d) At three points

Sol. (b) In $0 < x < 3$, only $x = 1, 2 \in Z$. That is, at only **two** points the given function $f(x) = [x]$ is not differentiable in $0 < x < 3$.
Recall that, a g.i.f. is not differentiable at all $x = c \in Z$.

09. $\int \frac{dx}{\sqrt{25 - 16x^2}}$ is equal to

(a) $\frac{1}{5}\sin^{-1} 4x + C$ (b) $\frac{1}{25}\sin^{-1} 16x + C$ (c) $\frac{1}{4}\sin^{-1}\frac{4x}{5} + C$ (d) $\frac{1}{16}\sin^{-1}\frac{4x}{5} + C$

Sol. (c) $\int \frac{dx}{\sqrt{25 - 16x^2}} = \int \frac{dx}{\sqrt{5^2 - (4x)^2}} = \frac{1}{4}\sin^{-1}\frac{4x}{5} + C$.

10. $\int_0^1 \frac{dx}{e^x + e^{-x}} = \tan^{-1} e + k$, then the value of k is

(a) e (b) $\frac{\pi}{4}$ (c) 0 (d) $-\frac{\pi}{4}$

Sol. (d) Let $I = \int_0^1 \frac{dx}{e^x + e^{-x}} = \int_0^1 \frac{e^x\, dx}{e^{2x} + 1}$

Put $e^x = t \Rightarrow e^x\, dx = dt$
When $x = 0 \Rightarrow t = 1$, when $x = 1 \Rightarrow t = e$.

$\therefore I = \int_1^e \frac{dt}{t^2 + 1} = \left[\tan^{-1} t\right]_1^e = \tan^{-1} e - \tan^{-1} 1 = \tan^{-1} e - \frac{\pi}{4}$ ...(i)

Comparing (i) with $\int_0^1 \frac{dx}{e^x + e^{-x}} = \tan^{-1} e + k$, we get $k = -\frac{\pi}{4}$.

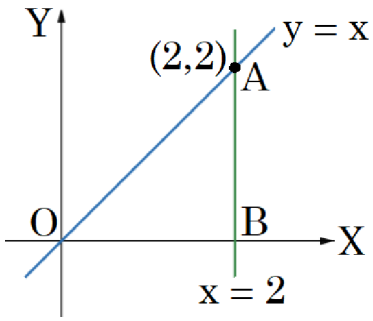
11. The area of the region bounded by the curve $y = x$ and x-axis, between $x = 0$ and $x = 2$ is

(a) 2 sq. units (b) $\frac{1}{2}$ sq. unit (c) 1 sq. unit (d) 4 sq. units

Sol. (a) Though we can solve this problem by integration.
But it is a MCQ, so we shall use the formula for area of Δ.

Therefore, the required area $= ar(\Delta OAB) = \frac{1}{2} \times OB \times AB$

$= \frac{1}{2} \times 2 \times 2 = 2$ sq. units.



Alternatively, by integration we can evaluate $\int_0^2 x\, dx$ to find the area of the region bounded by $y = x$, x-axis, between $x = 0$ and $x = 2$.

12. Product of the order and degree of differential equation $1 + \left(\frac{dy}{dx}\right)^3 = \lambda\left(\frac{d^3y}{dx^3}\right)^2$ is

(a) 5 (b) 6 (c) 2 (d) 3

Sol. (b) Order : 3, Degree : 2.
$\therefore$ Required product $= 3 \times 2 = 6$.

13. Which of the following is **not** a Linear Differential Equation?

(a) $(1+x^2)dy + 2xy\,dx = \cot x\,dx$ (b) $y + \frac{d}{dx}(xy) = x(\sin x + \log x)$

(c) $x(1+y^2)dx - y(1+x^2)dy = 0$ (d) $y\,dx - (x+3y^2)dy = 0$

Sol. (c) Only the D.E. given in option (c) $x(1+y^2)dx - y(1+x^2)dy = 0$ is **not** a Linear D.E.

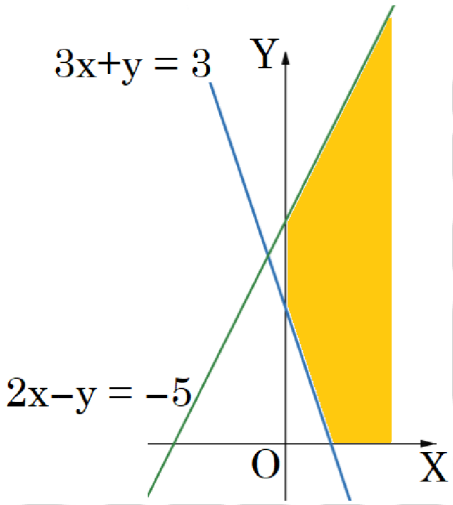
- Option (a) transforms to $\frac{dy}{dx} + \frac{2x}{1+x^2}y = \frac{\cot x}{1+x^2}$, which is Linear D.E. in y.
- Option (b) transforms to $\frac{dy}{dx} + \frac{2}{x}y = \sin x + \log x$, which is Linear D.E. in y.
- Option (d) transforms to $\frac{dx}{dy} - \frac{1}{y}x = 3y$, which is Linear D.E. in x.

14. The region represented by the system of inequations $3x + y \geq 3,\ 2x - y \geq -5;\ x,\ y \geq 0$ is

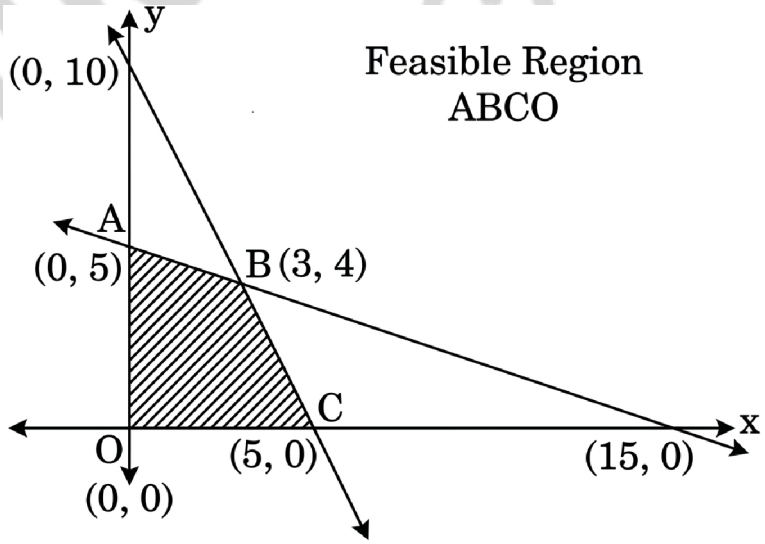
(a) unbounded in 1st quadrant (b) bounded in 1st quadrant

(c) unbounded in 2nd quadrant (d) bounded in 2nd quadrant

Sol. (a) The region represented by given system is "unbounded in 1st quadrant".



15. In the graph, the feasible region representing the Linear Programming Problem for maximizing objective function $Z = px + qy;\ p, q > 0$ is shaded. If all points on segment AB give max (Z), then which of the following is true?



(a) $p = 2q$ (b) $p = 3q$ (c) $q = 3p$ (d) $q = 2p$

Sol. (c) Since $\max(Z_A) = \max(Z_B)$

So, $p\times 0+q\times 5=p\times 3+q\times 4$

$\therefore q=3p$.

16. If $(3\hat{i}-2\hat{j}+5\hat{k})\times(4\hat{i}+p\hat{j}+q\hat{k})=\vec{0}$, then the values of p and q are

(a) $p=-\frac{2}{3}, q=\frac{5}{3}$ (b) $p=-\frac{8}{3}, q=\frac{20}{3}$ (c) $p=\frac{20}{3}, q=-\frac{8}{3}$ (d) $p=0, q=0$

Sol. (b) Since $(3\hat{i}-2\hat{j}+5\hat{k})\times(4\hat{i}+p\hat{j}+q\hat{k})=\vec{0}$ so, the non zero vectors involved must be parallel.

Therefore, $\frac{3}{4}=\frac{-2}{p}=\frac{5}{q}$

Consider $\frac{3}{4}=\frac{-2}{p}$ and $\frac{3}{4}=\frac{5}{q}$

$\therefore p=-\frac{8}{3}, q=\frac{20}{3}$.

17. Three points A(0, 1, 1), B(2, 0, –1) and C(1, 0, 3) form ΔABC. Then the ar(ΔABC) is

(a) $\frac{\sqrt{53}}{2}$ sq. units (b) $\sqrt{53}$ sq. units (c) $\frac{\sqrt{11}}{2}$ sq. units (d) $\sqrt{11}$ sq. units

Sol. (a) Here $\overrightarrow{AB}=2\hat{i}-\hat{j}-2\hat{k}$ and $\overrightarrow{AC}=\hat{i}-\hat{j}+2\hat{k}$.

$$\because \overrightarrow{AB}\times\overrightarrow{AC}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & -1 & -2\\ 1 & -1 & 2\end{vmatrix}=-4\hat{i}-6\hat{j}-\hat{k}$$

$$\therefore \text{ar}(\Delta ABC)=\frac{1}{2}\left|\overrightarrow{AB}\times\overrightarrow{AC}\right|=\frac{1}{2}\sqrt{(-4)^2+(-6)^2+(-1)^2}=\frac{1}{2}\sqrt{16+36+1}=\frac{\sqrt{53}}{2}\text{ sq. units}.$$

18. A box contains 4 red, 5 blue and 1 green marble. A child randomly takes out a marble from the box, notes down the colour and puts it back in the box. If the activity is repeated 3 times, what is the probability that at least one marble is red?

(a) $\frac{27}{125}$ (b) $\frac{8}{125}$ (c) $\frac{2}{125}$ (d) $\frac{98}{125}$

Sol. (d) Here $P(\text{red})=\frac{4}{10}=\frac{2}{5}$, $P(\text{not red})=1-\frac{2}{5}=\frac{3}{5}$.

$$\therefore P(\text{at least one red})=1-P(\text{no red in three trials})=1-\frac{3}{5}\times\frac{3}{5}\times\frac{3}{5}=1-\frac{27}{125}=\frac{98}{125}.$$

Direction : *Question numbers **19** and **20** are Assertion (A) and Reason (R) based questions carrying **1 mark** each. Two statements are given, one labeled Assertion (A) and other labeled Reason (R). Select the correct answer from the options given below.*

(a) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).
(b) Both Assertion (A) and Reason (R) are true and Reason (R) is **not** the correct explanation of Assertion (A).
(c) Assertion (A) is true but Reason (R) is false.
(d) Assertion (A) is false but Reason (R) is true.

19. **Assertion (A) :** If A and B are two square matrices such that AB and BA are defined, then it is not necessary that $AB=BA$.

Reason (R) : Product of two diagonal matrices of same order is commutative.

Sol. (b) Both Assertion and Reason are true, but Reason is not the correct explanation of Assertion.

20. **Assertion (A) :** A function $f:N\to N$ given by $f(x)=x^3+2, \forall\ x\in N$ is one-one but not onto.

Reason (R) : Since $\forall\ y\in N$ (Codomain), there does not exist $x=(y-2)^{1/3}$ in N (Domain) such that $f(x)=x^3+2=y$.

Sol. (a) Let $f(x_1) = f(x_2)$; $x_1, x_2 \in N$. Then $x_1^3 + 2 = x_2^3 + 2 \Rightarrow x_1^3 - x_2^3 = 0$

$\Rightarrow (x_1 - x_2)(x_1^2 + x_1x_2 + x_2^2) = 0$

Since $x_1, x_2 \in N$, $(x_1^2 + x_1x_2 + x_2^2) \neq 0 \because (x_1 - x_2) = 0 \Rightarrow x_1 = x_2$.

Clearly, f is one-one.

Let $y = 2 \in N$ such that $y = f(x) = x^3 + 2$. Then $2 = x^3 + 2 \Rightarrow x = 0 \notin N$.

That means, there are some y in the codomain whose pre-images are not in the domain.

Hence, f is not onto.

$\therefore$ Both Assertion and Reason are true and, Reason is the correct explanation of Assertion.

SECTION B

This section has 5 Very Short Answer questions of 2 marks each.

21. Evaluate $\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right) + \cot^{-1}\left(\frac{1}{\sqrt{3}}\right) + \tan^{-1}\left(\sin\left(-\frac{\pi}{2}\right)\right) + \tan^{-1}\left(\tan\frac{2\pi}{3}\right)$.

Sol. $\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right) + \cot^{-1}\left(\frac{1}{\sqrt{3}}\right) + \tan^{-1}\left(\sin\left(-\frac{\pi}{2}\right)\right) + \tan^{-1}\left(\tan\frac{2\pi}{3}\right)$

$= -\frac{\pi}{6} + \frac{\pi}{3} + \tan^{-1}(-1) + \tan^{-1}\left(\tan\left(\pi - \frac{\pi}{3}\right)\right)$

$= \frac{\pi}{6} - \frac{\pi}{4} + \tan^{-1}\left(-\tan\frac{\pi}{3}\right)$

$= -\frac{\pi}{12} - \tan^{-1}\left(\tan\frac{\pi}{3}\right) = -\frac{\pi}{12} - \frac{\pi}{3} = -\frac{5\pi}{12}$.

22. (a) Show that the function $f(x) = \begin{cases} \frac{\cos x}{-x + \frac{\pi}{2}}, & x \neq \frac{\pi}{2} \\ 1, & x = \frac{\pi}{2} \end{cases}$ is continuous at $x = \frac{\pi}{2}$.

OR

(b) Find whether the function $f(x) = \begin{cases} x - 1, & x < 2 \\ 2x - 3, & x \geq 2 \end{cases}$ at $x = 2$ is differentiable or not.

Sol. (a) Here $f\left(\frac{\pi}{2}\right) = 1$.

Also $\lim\limits_{x \to \frac{\pi}{2}} f(x) = \lim\limits_{x \to \frac{\pi}{2}} \frac{\cos x}{-x + \frac{\pi}{2}} = \lim\limits_{x \to \frac{\pi}{2}} \frac{\sin\left(-x + \frac{\pi}{2}\right)}{-x + \frac{\pi}{2}} = 1$

$\left[\because x \to \frac{\pi}{2} \ \therefore x - \frac{\pi}{2} \to 0 \text{ i.e., } -x + \frac{\pi}{2} \to 0\right.$

Since $f\left(\frac{\pi}{2}\right) = \lim\limits_{x \to \frac{\pi}{2}} f(x)$ $\therefore f(x)$ is continuous at $x = \frac{\pi}{2}$.

OR

(b) Here $f(2) = 2(2) - 3 = 1$.

LHD : $\lim\limits_{x \to 2^-} \frac{f(x) - f(2)}{x - 2} = \lim\limits_{x \to 2^-} \frac{(x-1) - (1)}{x - 2} = \lim\limits_{x \to 2^-} \frac{(x-2)}{x-2} = \lim\limits_{x \to 2^-} (1) = 1$

RHD : $\lim\limits_{x\to 2^+}\dfrac{f(x)-f(2)}{x-2}=\lim\limits_{x\to 2^+}\dfrac{(2x-3)-(1)}{x-2}=\lim\limits_{x\to 2^+}\dfrac{2(x-2)}{x-2}=\lim\limits_{x\to 2^+}(2)=2$

$\because$ LHD $\neq$ RHD $\therefore$ f is not differentiable at $x=2$.

23. Find the values of x for which $f(x)=x^x$, $x>0$ is increasing.

Sol. We have $f(x)=x^x$, $x>0$

$\Rightarrow f(x)=e^{\log x^x}$ $\Rightarrow f(x)=e^{x\log x}$

$\Rightarrow f'(x)=e^{x\log x}\left(x\times\dfrac{1}{x}+\log x\times 1\right)=x^x(1+\log x)$

For critical points, $f'(x)=x^x(1+\log x)=0$

$\Rightarrow 1+\log x=0$ $\Rightarrow \log x=-1$

$\therefore x=e^{-1}$

$\because f'(x)<0$ in $\left(0,\dfrac{1}{e}\right]$ $\therefore f(x)$ is decreasing in $\left(0,\dfrac{1}{e}\right]$

$\because f'(x)>0$ in $\left[\dfrac{1}{e},\infty\right)$ $\therefore f(x)$ is increasing in $\left[\dfrac{1}{e},\infty\right)$.

24. If the position vectors of three points A, B and C are $3\hat{i}+\hat{j}$, $5\hat{i}+6\hat{j}-3\hat{k}$ and $4\hat{j}$ respectively, then show that they form an isosceles triangle.

Sol. $\overrightarrow{AB}=\overrightarrow{OB}-\overrightarrow{OA}=(5\hat{i}+6\hat{j}-3\hat{k})-(3\hat{i}+\hat{j})=2\hat{i}+5\hat{j}-3\hat{k}$,

$\overrightarrow{BC}=\overrightarrow{OC}-\overrightarrow{OB}=(4\hat{j})-(5\hat{i}+6\hat{j}-3\hat{k})=-5\hat{i}-2\hat{j}+3\hat{k}$,

$\overrightarrow{CA}=\overrightarrow{OA}-\overrightarrow{OC}=(3\hat{i}+\hat{j})-(4\hat{j})=3\hat{i}-3\hat{j}$.

$\because \overrightarrow{AB}+\overrightarrow{BC}+\overrightarrow{CA}=2\hat{i}+5\hat{j}-3\hat{k}-5\hat{i}-2\hat{j}+3\hat{k}+3\hat{i}-3\hat{j}=\vec{0}$

So, the points A, B and C form a triangle.

Now $|\overrightarrow{AB}|=\sqrt{2^2+5^2+(-3)^2}=\sqrt{4+25+9}=\sqrt{38}$,

$|\overrightarrow{BC}|=\sqrt{(-5)^2+(-2)^2+3^2}=\sqrt{25+4+9}=\sqrt{38}$ and $|\overrightarrow{CA}|=\sqrt{3^2+(-3)^2}=\sqrt{9+9}=\sqrt{18}$

As $|\overrightarrow{AB}|=|\overrightarrow{BC}|$, the triangle has two equal sides.

Therefore, the points A, B and C form an isosceles triangle.

25. (a) Let two rods placed on the ground be represented by vectors $4\hat{i}-\hat{j}+3\hat{k}$ and $-2\hat{i}+\hat{j}-2\hat{k}$. Find a vector representing a flag-post of height 5 m that has to be erected perpendicular to both the rods.

OR

(b) A unit vector $\vec{a}$ is such that it makes an angle $\dfrac{\pi}{4}$ with x-axis, $\dfrac{\pi}{3}$ with y-axis and an acute angle θ with z-axis. Find θ and the components of $\vec{a}$.

Sol. (a) Let $\vec{a}=4\hat{i}-\hat{j}+3\hat{k}$, $\vec{b}=-2\hat{i}+\hat{j}-2\hat{k}$.

Now $\vec{a}\times\vec{b}=\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\4&-1&3\\-2&1&-2\end{vmatrix}=-\hat{i}+2\hat{j}+2\hat{k}=\vec{p}$ *say.*

Since the required vector is perpendicular to $\vec{a}$ and $\vec{b}$ both.

So, it must be in the direction of $\vec{a}\times\vec{b}$.

Note that, the flag-post is of height 5 m i.e., magnitude of vector should be same as this height.

Hence, required vector of magnitude 5 m is $= 5\hat{p} = 5\dfrac{\vec{p}}{|\vec{p}|} = 5\left(\dfrac{-\hat{i}+2\hat{j}+2\hat{k}}{\sqrt{1+4+4}}\right) = \dfrac{5(-\hat{i}+2\hat{j}+2\hat{k})}{3}$.

OR

(b) Given $\alpha = \dfrac{\pi}{4}, \beta = \dfrac{\pi}{3}, \gamma = \theta$.

Since $\cos^2\alpha + \cos^2\beta + \cos^2\theta = 1$. So, $\cos^2\dfrac{\pi}{4} + \cos^2\dfrac{\pi}{3} + \cos^2\theta = 1$.

$\Rightarrow \left(\dfrac{1}{\sqrt{2}}\right)^2 + \left(\dfrac{1}{2}\right)^2 + \cos^2\theta = 1$

$\Rightarrow \dfrac{1}{2} + \dfrac{1}{4} + \cos^2\theta = 1$

$\Rightarrow \cos^2\theta = 1 - \dfrac{3}{4} = \dfrac{1}{4}$

$\Rightarrow \cos\theta = \dfrac{1}{2}$ [As θ is acute angle

$\Rightarrow \theta = \dfrac{\pi}{3}$.

Therefore, $\vec{a} = \dfrac{1}{\sqrt{2}}\hat{i} + \dfrac{1}{2}\hat{j} + \dfrac{1}{2}\hat{k}$.

Hence, the components of $\vec{a}$ along x, y and z axes are $\dfrac{1}{\sqrt{2}}, \dfrac{1}{2}, \dfrac{1}{2}$ respectively.

SECTION C

There are 6 Short Answer questions in this section. Each is of 3 marks.

26. (a) Let $A = \mathbb{R} - \{3\}$ and $B = \mathbb{R} - \{1\}$. A function $f: A \to B$ is defined by $f(x) = \left(\dfrac{x-2}{x-3}\right)$. Find whether f is one-one and onto.

OR

(b) Let n be a fixed positive integer. A relation R is defined in set Z such that $R = \{(x, y): (x - y)$ is divisible by $n;\ x, y \in Z\}$. Determine if R is an equivalence relation.

Sol. (a) Let $f(x_1) = f(x_2)$ for some $x_1, x_2 \in A$ (domain).

Then $\dfrac{x_1 - 2}{x_1 - 3} = \dfrac{x_2 - 2}{x_2 - 3}$

$\Rightarrow x_1x_2 - 3x_1 - 2x_2 + 6 = x_1x_2 - 2x_1 - 3x_2 + 6$

$\Rightarrow -x_1 = -x_2$

$\Rightarrow x_1 = x_2$

$\therefore$ f is one-one.

Let $y = f(x),\ y \in B = \mathbb{R} - \{1\}$.

That is, $y = \dfrac{x-2}{x-3}$ $\Rightarrow xy - 3y = x - 2$

$\Rightarrow 2 - 3y = x - xy$

$\Rightarrow 2 - 3y = x(1 - y)$

$\Rightarrow x = \dfrac{2-3y}{1-y}$

Since $y \in B$, $y \neq 1$, so x is always defined.
For every $y \in B$, there exists $x \in A$, hence f is onto.

OR

(b) $R = \{(x, y) : (x - y)$ is divisible by $n;\ x, y \in Z$
For any $a \in Z$, $a - a = 0 \Rightarrow a - a$ is divisible by n so, (a, a) lies in R.
$\therefore$ R is reflexive.
Also for $(a, b) \in R$ implies that $a - b$ is divisible by n.
That is, $a - b = np$ for some $p \in Z \quad \Rightarrow b - a = n(-p)$
$\Rightarrow b - a$ is divisible by n $\therefore (b, a) \in R$.
So, R is symmetric.
Now for some $a, b, c \in Z$ such that $(a, b) \in R$ and $(b, c) \in R$.
Then $(a, b) \in R$ implies that $a - b$ is divisible by n.
That is, $a - b = np$ for some $p \in Z$.
Also $(b, c) \in R$ implies that $b - c$ is divisible by n.
That is, $b - c = nq$ for some $q \in Z$.
$\therefore (a - b) + (b - c) = np + nq \Rightarrow a - c = n(p + q)$.
That means, a – c is divisible by n.
So, $(a, c) \in R$.
Hence, R is transitive.
$\therefore$ R is an equivalence relation on the set of all integers.

27. If $A = \begin{bmatrix} 3 & 2 & 0 \\ 1 & 4 & 0 \\ 0 & 0 & 5 \end{bmatrix}$, then compute $A^2 - 7A + 10I$.

Sol. $A^2 = AA = \begin{bmatrix} 3 & 2 & 0 \\ 1 & 4 & 0 \\ 0 & 0 & 5 \end{bmatrix}\begin{bmatrix} 3 & 2 & 0 \\ 1 & 4 & 0 \\ 0 & 0 & 5 \end{bmatrix} = \begin{bmatrix} 11 & 14 & 0 \\ 7 & 18 & 0 \\ 0 & 0 & 25 \end{bmatrix}$.

Now $A^2 - 7A + 10I = \begin{bmatrix} 11 & 14 & 0 \\ 7 & 18 & 0 \\ 0 & 0 & 25 \end{bmatrix} - 7\begin{bmatrix} 3 & 2 & 0 \\ 1 & 4 & 0 \\ 0 & 0 & 5 \end{bmatrix} + 10\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 11 & 14 & 0 \\ 7 & 18 & 0 \\ 0 & 0 & 25 \end{bmatrix} - \begin{bmatrix} 21 & 14 & 0 \\ 7 & 28 & 0 \\ 0 & 0 & 35 \end{bmatrix} + \begin{bmatrix} 10 & 0 & 0 \\ 0 & 10 & 0 \\ 0 & 0 & 10 \end{bmatrix}$

$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = O$.

28. (a) If $xy = e^{x-y}$, then find $\frac{dy}{dx}$.

OR

(b) Differentiate $\tan^{-1}\left(\frac{\sqrt{1+x^2} - \sqrt{1-x^2}}{\sqrt{1+x^2} + \sqrt{1-x^2}}\right)$ with respect to $\cos^{-1} x^2$.

Sol. (a) $xy = e^{x-y}$

Taking log both the sides, we get $\log(xy) = \log e^{x-y}$

$\Rightarrow \log x + \log y = (x-y)\log e$

$\because \log e = 1 \qquad \therefore \log x + \log y = (x-y)$

Now on differentiating w.r.t. x, $\frac{1}{x} + \frac{1}{y} \times \frac{dy}{dx} = 1 - \frac{dy}{dx}$

$\Rightarrow \frac{1}{y} \times \frac{dy}{dx} + \frac{dy}{dx} = 1 - \frac{1}{x}$

$\Rightarrow \left(\frac{1}{y} + 1\right)\frac{dy}{dx} = \frac{x-1}{x}$

$\Rightarrow \left(\frac{1+y}{y}\right)\frac{dy}{dx} = \frac{x-1}{x}$

$\therefore \frac{dy}{dx} = \frac{y(x-1)}{x(1+y)}$.

OR

(b) Let $y = \tan^{-1}\left(\frac{\sqrt{1+x^2} - \sqrt{1-x^2}}{\sqrt{1+x^2} + \sqrt{1-x^2}}\right)$ and $z = \cos^{-1} x^2$.

Clearly, $\cos z = x^2$.

Put $x^2 = \cos z$ in y.

$\therefore y = \tan^{-1}\left(\frac{\sqrt{1+\cos z} - \sqrt{1-\cos z}}{\sqrt{1+\cos z} + \sqrt{1-\cos z}}\right)$

$\Rightarrow y = \tan^{-1}\left(\frac{\sqrt{2\cos^2\frac{z}{2}} - \sqrt{2\sin^2\frac{z}{2}}}{\sqrt{2\cos^2\frac{z}{2}} + \sqrt{2\sin^2\frac{z}{2}}}\right)$

$\Rightarrow y = \tan^{-1}\left(\frac{\cos\frac{z}{2} - \sin\frac{z}{2}}{\cos\frac{z}{2} + \sin\frac{z}{2}}\right) = \tan^{-1}\left(\frac{1 - \tan\frac{z}{2}}{1 + \tan\frac{z}{2}}\right)$

$\Rightarrow y = \tan^{-1}\left(\frac{\tan\frac{\pi}{4} - \tan\frac{z}{2}}{1 + \tan\frac{\pi}{4}\tan\frac{z}{2}}\right)$

$\Rightarrow y = \tan^{-1}\tan\left(\frac{\pi}{4} - \frac{z}{2}\right)$

$\Rightarrow y = \left(\frac{\pi}{4} - \frac{z}{2}\right)$

On differentiating with respect to z both the sides, we get $\frac{dy}{dz} = 0 - \frac{1}{2} = -\frac{1}{2}$.

29. Solve the following Linear Programming Problem graphically.

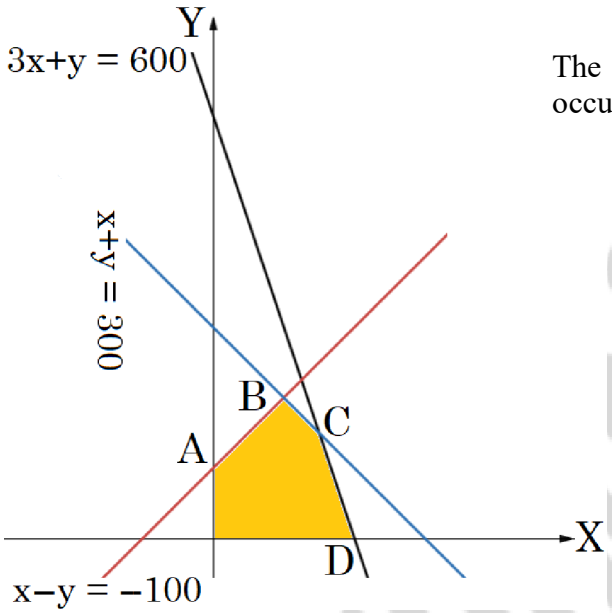
Maximise $Z = 200x + 120y$

Subject to the constraints $x+y\le 300,\ 3x+y\le 600,\ x-y\ge -100;\ x, y\ge 0$.

Sol. Consider the graph shown below.

Corner points are O(0, 0), A(0, 100), B(100, 200), C(150, 150) and D(200, 0).

$\therefore Z_O = 0,\ Z_A = 12000,\ Z_B = 44000,\ Z_C = 48000,\ Z_D = 40000$



The maximum value of Z is 48000, which occurs at the point (150, 150).

30. (a) Find a point on the line $\frac{x-2}{3}=\frac{1-y}{2}=\frac{z-3}{2}$ at a distance of $\sqrt{2}$ units from the point $(1, 2, 3)$.

OR

(b) Find the shortest distance between the lines

$\vec{r}=(4+\lambda)\hat{i}+(2\lambda-1)\hat{j}-3\lambda\hat{k}$

$\vec{r}=(1+2\mu)\hat{i}+(2-5\mu)\hat{k}+(4\mu-1)\hat{j}$.

Sol. (a) Let $A(1, 2, 3)$.

Let $\frac{x-2}{3}=\frac{1-y}{2}=\frac{z-3}{2}=\lambda$.

The coordinates of any random point on the line is $P(3\lambda+2, 1-2\lambda, 2\lambda+3)$.

Taking $AP=\sqrt{2}$ i.e., $AP^2=2$

$\Rightarrow (3\lambda+2-1)^2+(1-2\lambda-2)^2+(2\lambda+3-3)^2=2$

$\Rightarrow (3\lambda+1)^2+(-1-2\lambda)^2+(2\lambda)^2=2$

$\Rightarrow 9\lambda^2+6\lambda+1+1+4\lambda+4\lambda^2+4\lambda^2=2$

$\Rightarrow 17\lambda^2+10\lambda=0$

$\Rightarrow \lambda(17\lambda+10)=0$

Either $\lambda=0$ or $(17\lambda+10)=0$

Either $\lambda=0$ or $\lambda=-\frac{10}{17}$.

When $\lambda=0$: Required point is $(2, 1, 3)$

When $\lambda=-\frac{10}{17}$: Required point is $\left(\frac{4}{17}, \frac{37}{17}, \frac{31}{17}\right)$.

OR

(b) Rewriting the lines, we get $\vec{r} = 4\hat{i} - \hat{j} + \lambda(\hat{i} + 2\hat{j} - 3\hat{k})$, $\vec{r} = \hat{i} - \hat{j} + 2\hat{k} + \mu(2\hat{i} + 4\hat{j} - 5\hat{k})$.

So, $\vec{a}_1 = 4\hat{i} - \hat{j}$, $\vec{b}_1 = \hat{i} + 2\hat{j} - 3\hat{k}$; $\vec{a}_2 = \hat{i} - \hat{j} + 2\hat{k}$, $\vec{b}_2 = 2\hat{i} + 4\hat{j} - 5\hat{k}$.

Now $\vec{a}_2 - \vec{a}_1 = -3\hat{i} + 2\hat{k}$.

Also $\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -3 \\ 2 & 4 & -5 \end{vmatrix} = 2\hat{i} - \hat{j}$.

Therefore, S.D. $= \dfrac{|(\vec{a}_2 - \vec{a}_1).(\vec{b}_1 \times \vec{b}_2)|}{|(\vec{b}_1 \times \vec{b}_2)|} = \dfrac{|(-3\hat{i} + 2\hat{k}).(2\hat{i} - \hat{j})|}{\sqrt{2^2 + (-1)^2 + 0^2}} = \dfrac{|-6|}{\sqrt{4+1}} = \dfrac{6}{\sqrt{5}}$ units.

31. In a school, the probability of holding a debate competition is $\frac{1}{3}$ and that of a quiz competition is $\frac{2}{3}$. In the two participating teams, A has 4 girls and 6 boys and B has 7 girls and 3 boys. If a debate competition is held, the students are selected from team A and for the quiz competition they are selected from team B. If only two students are to be chosen from the teams, then find the probability that one will be a girl and the other a boy.

Sol. Let E_1 and E_2 be the respective events that a Debate competition is held and that a Quiz competition is held. Also let A be the event that one girl and one boy are selected.

$\therefore P(E_1) = \frac{1}{3}$, $P(E_2) = \frac{2}{3}$, $P(A \mid E_1) = \dfrac{{}^4C_1 \times {}^6C_1}{{}^{10}C_2} = \dfrac{4 \times 6}{45} = \dfrac{24}{45}$, $P(A \mid E_2) = \dfrac{{}^7C_1 \times {}^3C_1}{{}^{10}C_2} = \dfrac{7 \times 3}{45} = \dfrac{21}{45}$.

Using total probability, $P(A) = P(E_1).P(A \mid E_1) + P(E_2).P(A \mid E_2)$

$\Rightarrow P(A) = \frac{1}{3} \times \frac{24}{45} + \frac{2}{3} \times \frac{21}{45} = \frac{24+42}{135} = \frac{66}{135} = \frac{22}{45}$.

SECTION D

There are 4 Long Answer questions in this section. Each is of 5 marks.

32. (a) Find $\int \dfrac{x}{(x-1)(x^2+4)}\,dx$.

OR

(b) Evaluate $\int_0^1 \dfrac{x \tan^{-1} x}{(1+x^2)^{3/2}}\,dx$.

Sol. (a) Consider $\dfrac{x}{(x-1)(x^2+4)} = \dfrac{A}{x-1} + \dfrac{B(2x)+C}{x^2+4}$

$\Rightarrow x = A(x^2+4) + B(2x)(x-1) + C(x-1)$

On comparing the like terms on both the sides, we get $A = \frac{1}{5}$, $B = -\frac{1}{10}$, $C = \frac{4}{5}$.

$$\therefore \int \frac{x}{(x-1)(x^2+4)}\,dx = \int \left[\frac{1}{5(x-1)} - \frac{1}{10} \times \frac{2x}{x^2+4} + \frac{4}{5} \times \frac{1}{x^2+4} \right] dx$$

$$= \frac{1}{5}\log|x-1| - \frac{1}{10} \times \log|x^2+4| + \frac{4}{5} \times \frac{1}{2} \tan^{-1}\frac{x}{2} + C$$

$$= \frac{1}{5}\log|x-1| - \frac{1}{10}\log|x^2+4| + \frac{2}{5}\tan^{-1}\frac{x}{2} + C$$

Recall that, $\int \frac{f'(x)}{f(x)}dx = \log|f(x)| + C$.

OR

(b) Put $\tan^{-1}x = \theta \Rightarrow \frac{1}{1+x^2}dx = d\theta \Rightarrow dx = (1+\tan^2\theta)d\theta = \sec^2\theta\, d\theta$. Also $x = \tan\theta$.

When $x = 0 \Rightarrow \theta = 0$, when $x = 1 \Rightarrow \theta = \frac{\pi}{4}$.

$$\therefore \int_0^1 \frac{x\tan^{-1}x}{(1+x^2)^{3/2}}dx = \int_0^{\frac{\pi}{4}} \frac{\theta\tan\theta}{(1+\tan^2\theta)^{3/2}} \times \sec^2\theta\, d\theta$$

$$= \int_0^{\frac{\pi}{4}} \frac{\theta\tan\theta}{\sec^3\theta} \times \sec^2\theta\, d\theta = \int_0^{\frac{\pi}{4}} \frac{\theta\tan\theta}{\sec\theta}\, d\theta = \int_0^{\frac{\pi}{4}} \theta\sin\theta\, d\theta \quad \ldots(i)$$

Consider $\int \theta\sin\theta\, d\theta = \theta\int \sin\theta\, d\theta - \int\left[\frac{d}{d\theta}(\theta)\int \sin\theta\, d\theta\right]d\theta$

$$= -\theta\cos\theta + \int \cos\theta d\theta = -\theta\cos\theta + \sin\theta$$

By (i), $\int_0^1 \frac{x\tan^{-1}x}{(1+x^2)^{3/2}}dx = \left[-\theta\cos\theta + \sin\theta\right]_0^{\pi/4}$

$$= \left[-\frac{\pi}{4}\cos\frac{\pi}{4} + \sin\frac{\pi}{4}\right] - \left[-0\cos 0 + \sin 0\right]$$

$$= -\frac{\pi}{4}\times\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}$$

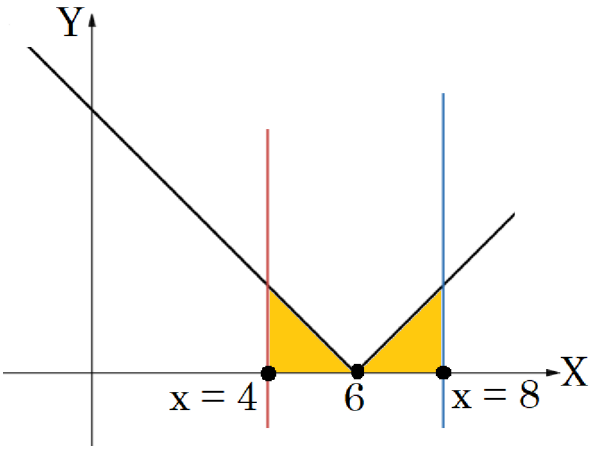
$$= \left(1 - \frac{\pi}{4}\right)\times\frac{1}{\sqrt{2}}$$

$$= \left(\frac{4-\pi}{4\sqrt{2}}\right).$$

33. Using integration, find the area of the region enclosed by the curve $y = |x-6|$, the x-axis, and between $x = 4$ and $x = 8$.

Sol. We need to find the area enclosed by $y = |x-6|$, the x-axis, and between $x = 4$ and $x = 8$.

$$\because y = |x-6| = \begin{cases} x-6, & x \geq 6 \\ -(x-6), & x < 6 \end{cases}.$$



$$\text{Required area} = \int_4^6 |x-6|dx + \int_6^8 |x-6|dx$$

$$= -\int_4^6 (x-6)dx + \int_6^8 (x-6)dx$$

$$= -\frac{1}{2}\left[(x-6)^2\right]_4^6 + \frac{1}{2}\left[(x-6)^2\right]_6^8$$

$$= -\frac{1}{2}[0-4] + \frac{1}{2}[4-0]$$

$= 2 + 2 = 4$ Sq. units.

34. (a) Solve the differential equation $y e^y dx = (y^3 + 2x e^y)dy$ when $y(0) = 1$.

OR

(b) Find the general solution of the differential equation $(x^3 - 3xy^2)dx = (y^3 - 3x^2y)dy$.

Sol. (a) Rewriting the D.E., $\frac{dx}{dy} = \frac{y^3 + 2x e^y}{y e^y}$

$\Rightarrow \frac{dx}{dy} + \left(-\frac{2}{y}\right)x = \frac{y^2}{e^y}$

On comparing with $\frac{dx}{dy} + P(y)x = Q(y)$, we get $P(y) = -\frac{2}{y}$, $Q(y) = \frac{y^2}{e^y}$

So, I.F. $= e^{\int -\frac{2}{y}dy} = e^{-2\log y} = \frac{1}{e^{\log y^2}} = \frac{1}{y^2}$.

The general solution is given by $\frac{1}{y^2} \times x = \int \frac{1}{y^2} \times \frac{y^2}{e^y} dy + C$

$\Rightarrow \frac{x}{y^2} = \int e^{-y} dy + C$

$\Rightarrow \frac{x}{y^2} = \frac{e^{-y}}{-1} + C$

$\Rightarrow \frac{x}{y^2} + e^{-y} = C$

As $y(0) = 1$ i.e., $y = 1, x = 0$.

So, $\frac{0}{1^2} + e^{-1} = C \quad \Rightarrow C = e^{-1}$.

Hence, the required solution is $\frac{x}{y^2} + e^{-y} = e^{-1}$ or $x = y^2\left(\frac{1}{e} - e^{-y}\right)$.

OR

(b) Rewriting the D.E., $\frac{dy}{dx} = \frac{x^3 - 3xy^2}{y^3 - 3x^2y}$

Put $y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$

$\therefore v + x\frac{dv}{dx} = \frac{x^3 - 3x^3v^2}{v^3x^3 - 3x^3v}$

$\Rightarrow x\frac{dv}{dx} = \frac{1 - 3v^2}{v^3 - 3v} - v$

$\Rightarrow x\frac{dv}{dx} = \frac{1 - v^4}{v^3 - 3v}$

$\Rightarrow \int \frac{v^3 - 3v}{v^4 - 1} dv = -\int \frac{dx}{x}$

$\Rightarrow \frac{1}{4}\int \frac{4v^3}{v^4 - 1} dv - 3\int \frac{v}{(v^2)^2 - 1} dv = -\log|x|$ [Put $v^2 = t$ in second integral of LHS

$$\therefore \frac{1}{4}\log|v^4-1|-\frac{3}{2}\int\frac{dt}{t^2-1}=-\log|x|$$

$$\Rightarrow \frac{1}{4}\log|y^4-x^4|-\log|x|-\frac{3}{2}\times\frac{1}{2}\log\left|\frac{v^2-1}{v^2+1}\right|=-\log|x|+\log|C|$$

$$\Rightarrow \frac{1}{4}\log|y^4-x^4|-\frac{3}{4}\log\left|\frac{y^2-x^2}{y^2+x^2}\right|=\log|C|$$

$$\Rightarrow \log|y^4-x^4|-3\log\left|\frac{y^2-x^2}{y^2+x^2}\right|=\log|C^4|$$

$$\Rightarrow \log|(y^2-x^2)(y^2+x^2)|+\log\left|\left(\frac{y^2+x^2}{y^2-x^2}\right)^3\right|=\log|C^4|$$

$$\Rightarrow \log\left|(y^2-x^2)(y^2+x^2)\times\left(\frac{y^2+x^2}{y^2-x^2}\right)^3\right|=\log|C^4|$$

$$\Rightarrow \log\left|\frac{(y^2+x^2)^4}{(y^2-x^2)^2}\right|=\log|C^4|$$

$$\Rightarrow 2\log\left|\frac{(y^2+x^2)^2}{(y^2-x^2)}\right|=2\log|C^2|$$

$$\Rightarrow \frac{(y^2+x^2)^2}{(y^2-x^2)}=\lambda, \text{ where } \lambda=C^2$$

$\therefore (y^2+x^2)^2=\lambda(y^2-x^2)$ is the required solution.

35. Find the equation of a line (in vector and Cartesian form) that passes through the point of intersection of lines $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$ and $\frac{x-4}{5}=\frac{y-1}{2}=z$ and is parallel to the vector $3\hat{i}+2\hat{j}-8\hat{k}$.

Sol. Let $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}=\lambda$ and $\frac{x-4}{5}=\frac{y-1}{2}=z=\mu$.

The coordinates of any random point on both the lines are $P(2\lambda+1,3\lambda+2,4\lambda+3)$ and $Q(5\mu+4,2\mu+1,\mu)$ respectively.

When the lines intersect, the points P and Q will coincide.

That is, $2\lambda+1=5\mu+4$ ...(i)

$3\lambda+2=2\mu+1$ ...(ii)

$\mu=4\lambda+3$ ...(iii)

Solving (i) and (iii), we get $\lambda=-1, \mu=-1$

Put $\lambda=-1, \mu=-1$ in (ii).

LHS of (ii) : $3\lambda+2=3(-1)+2=-1$;

RHS of (ii) : $2\mu+1=2(-1)+1=-1$.

Hence, the points P and Q coincide and therefore the point of intersection is (–1, –1, –1).

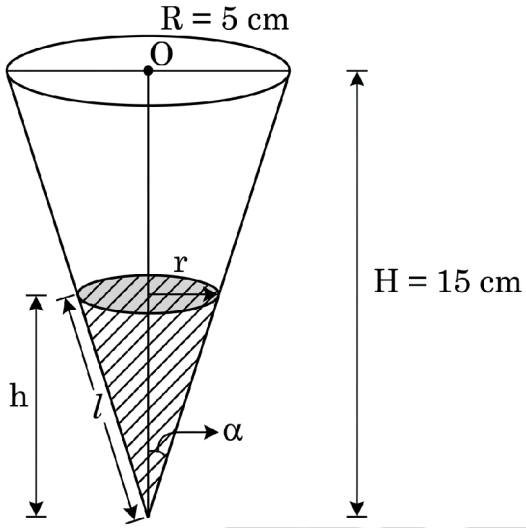
Now vector equation of a line passing through (–1, –1, –1) and parallel to $3\hat{i}+2\hat{j}-8\hat{k}$ is given by $\vec{r}=-\hat{i}-\hat{j}-\hat{k}+\lambda(3\hat{i}+2\hat{j}-8\hat{k})$

Also the Cartesian equation of line will be $\frac{x+1}{3}=\frac{y+1}{2}=\frac{z+1}{-8}$.

SECTION E

In this section there are 3 Case Study questions of 4 marks each.

36. At a birthday party, children are being served orange juice in conical cups, as shown in the figure.



Each cup is 15 cm deep and has a radius 5 cm. The juice is being poured into this cup at a rate of $0.1\ \text{cm}^3/\text{s}$.

Based on the above information, answer the following questions.

(i) Establish a relation between the height h of the juice in the cup and radius r of the surface of the juice in the cup, if the semi-vertical angle of the cone is α.

(ii) At what rate is the juice level in the cup rising when the juice is 6 cm deep?

(iii) (a) When the juice is 6 cm deep, then find at what rate is the upper surface area of juice increasing?

OR

(iii) (b) When the juice is 6 cm deep, then find the rate at which the wetted surface area of the cup is increasing.

Sol. **(i)** In the right-angled triangle formed by the semi-vertical angle α, $\tan\alpha = \frac{r}{h}$.

Also from the full cup, $\tan\alpha = \frac{R}{H} = \frac{5}{15} = \frac{1}{3}$.

$\therefore \frac{1}{3} = \frac{r}{h} \Rightarrow h = 3r$.

(ii) Volume of the cone, $V = \frac{\pi}{3}r^2h = \frac{\pi}{3}\left(\frac{h}{3}\right)^2 h = \frac{\pi h^3}{27}$

$$\Rightarrow \frac{dV}{dt} = \frac{3\pi h^2}{27} \times \frac{dh}{dt}$$

$$\Rightarrow \frac{dh}{dt} = 0.1 \times \frac{9}{\pi h^2}$$

$$\Rightarrow \left(\frac{dh}{dt}\right)_{\text{at h=6 cm}} = 0.1 \times \frac{9}{\pi \times 6^2} = \frac{1}{40\pi}\ \text{cm/s}.$$

(iii) (a) The upper surface area is $A = \pi r^2 = \pi\left(\frac{h}{3}\right)^2$

$$\Rightarrow A=\frac{\pi}{9}\times h^2$$

$$\Rightarrow \frac{dA}{dt}=\frac{2\pi}{9}\times h\times\frac{dh}{dt}$$

$$\Rightarrow\left(\frac{dA}{dt}\right)_{\text{at h=6 cm}}=\frac{2\pi}{9}\times 6\times\frac{1}{40\pi}=\frac{1}{30}\ \text{cm}^2/\text{s}.$$

OR

(iii) (b) Slant height $(l)=\sqrt{r^2+h^2}=\sqrt{\left(\frac{h}{3}\right)^2+h^2}=\sqrt{\frac{h^2}{9}+h^2}=\sqrt{\frac{10h^2}{9}}=\frac{h\sqrt{10}}{3}$.

The wetted (curved) surface area is $A=\pi r l=\pi\frac{h}{3}\times\frac{\sqrt{10}\,h}{3}=\frac{\pi\sqrt{10}\,h^2}{9}$

$$\Rightarrow \frac{dA}{dt}=\frac{2\pi\sqrt{10}\,h}{9}\times\frac{dh}{dt}$$

$$\Rightarrow\left(\frac{dA}{dt}\right)_{\text{at h=6 cm}}=\frac{2\pi\sqrt{10}\times 6}{9}\times\frac{1}{40\pi}=\frac{\sqrt{10}}{30}\ \text{cm}^2/\text{s}.$$

37. A carpenter needs to design a wooden box in the shape of a cuboid such that the sum of its length and breadth is 3 cm more than its height. Twice of its length, thrice of its breadth and its height add up to 10 cm. Its breadth added to 7 times its height is 1 cm less than 3 times its length.
Based on the given information, answer the following questions.
(i) Write the equations representing the various dimensions and express them as the matrix equation $AX=B$.
(ii) Find if A^{-1} exists. Justify your answer.
(iii) (a) Find A^{-1}.

OR

(iii) (b) Find A^2+7I.

Sol. Let the length, breadth, and height of the box be x, y and z respectively (in cm).
(i) According to the question, we get $x+y-z=3,\ 2x+3y+z=10,\ 3x-y-7z=1$

$$\therefore\begin{bmatrix}1&1&-1\\2&3&1\\3&-1&-7\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=\begin{bmatrix}3\\10\\1\end{bmatrix},\ \text{where } A=\begin{bmatrix}1&1&-1\\2&3&1\\3&-1&-7\end{bmatrix},\ X=\begin{bmatrix}x\\y\\z\end{bmatrix},\ B=\begin{bmatrix}3\\10\\1\end{bmatrix}.$$

(ii) $\because |A|=1(-21+1)-1(-14-3)+(-1)(-2-9)=-20+17+11=8\neq 0$

$\therefore A^{-1}$ exists.

(iii) (a) We shall use cofactors of elements a_{ij} to write the adjoint of matrix A.

$$\because \text{adj.A}=\begin{bmatrix}-20&8&4\\17&-4&-3\\-11&4&1\end{bmatrix}\qquad\therefore A^{-1}=\frac{\text{adj.A}}{\det.(A)}=\frac{1}{8}\begin{bmatrix}-20&8&4\\17&-4&-3\\-11&4&1\end{bmatrix}.$$

OR

(iii) (b) $A^2=AA=\begin{bmatrix}1&1&-1\\2&3&1\\3&-1&-7\end{bmatrix}\begin{bmatrix}1&1&-1\\2&3&1\\3&-1&-7\end{bmatrix}=\begin{bmatrix}0&5&7\\11&10&-6\\-20&7&45\end{bmatrix}$

$$\therefore A^2+7I=\begin{bmatrix}0&5&7\\11&10&-6\\-20&7&45\end{bmatrix}+\begin{bmatrix}7&0&0\\0&7&0\\0&0&7\end{bmatrix}=\begin{bmatrix}7&5&7\\11&17&-6\\-20&7&52\end{bmatrix}.$$

38. An NGO organizes a charity event in which they decide to distribute woolen caps to protect children from winter. The caps to be distributed are in three separate boxes, Box I has 30 red caps, Box II has 20 red and 10 green caps, and Box III has 30 green caps. The probability that a Box i is selected and a cap picked out is $\frac{i}{6}$, where $i=1, 2, 3$.

Based on the above information, answer the following questions.
A person selects a cap.
(i) What is the probability that he selects a red cap?
(ii) If he selects a green cap, what is the probability that the cap has come from Box II?

Sol. Box I: 30 Red caps; Box II: 20 Red and 10 Green caps (Total 30); Box III: 30 Green caps.

Now the probabilities of selecting Box i: $P(E_1)=\frac{1}{6}$, $P(E_2)=\frac{2}{6}$, $P(E_3)=\frac{3}{6}$.

(i) Let R be the event of picking a Red cap.

So, $P(R)=P(E_1)P(R\mid E_1)+P(E_2)P(R\mid E_2)+P(E_3)P(R\mid E_3)$

$$\Rightarrow P(R)=\frac{1}{6}\times\frac{30}{30}+\frac{2}{6}\times\frac{20}{30}+\frac{3}{6}\times 0=\frac{70}{6\times 30}=\frac{7}{18}$$

(ii) Let G be the event of picking a Green cap.

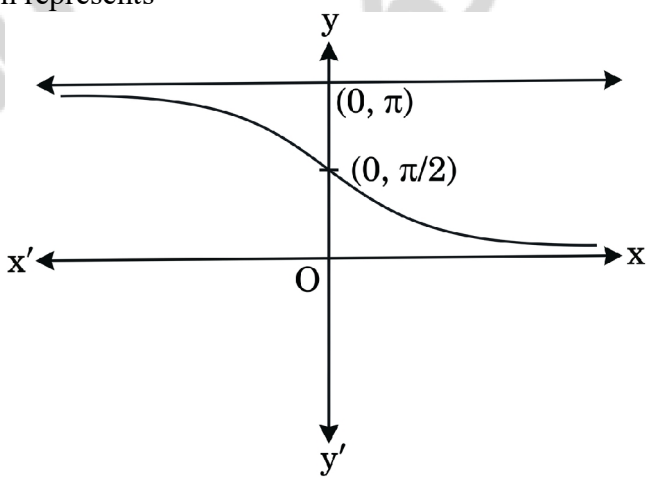
So, $P(G)=1-P(R)=1-\frac{7}{18}=\frac{11}{18}$.

Using Bayes' theorem, $P(E_2\mid G)=\frac{P(E_2)\times P(G\mid E_2)}{P(G)}$

$$\Rightarrow P(E_2\mid G)=\frac{\frac{2}{6}\times\frac{10}{30}}{\frac{11}{18}}=\frac{2}{11}.$$

☑ Q.P. Code - 65/4/2

01. The following graph represents



(a) $y=\sec^{-1}x$ (b) $y=\cot^{-1}x$ (c) $y=\tan^{-1}x$ (d) $y=\operatorname{cosec}^{-1}x$

Sol. (b) The graph is of $y=\cot^{-1}x$. Since the domain is $x\in(-\infty,\infty)$ and range is $y\in(0,\pi)$.

02. Let $A=[a_{ij}]$ be a 2×2 matrix whose elements are given by $a_{ij}=\frac{(2i-j)^2}{3}$. Then A' is

(a) $\begin{bmatrix}\frac{1}{3} & 3\\ 0 & \frac{4}{3}\end{bmatrix}$ (b) $\begin{bmatrix}\frac{1}{3} & 0\\ 3 & \frac{4}{3}\end{bmatrix}$ (c) $\begin{bmatrix}\frac{4}{3} & 3\\ 0 & \frac{1}{3}\end{bmatrix}$ (d) $\begin{bmatrix}\frac{4}{3} & 0\\ 3 & \frac{1}{3}\end{bmatrix}$

Sol. (a) Since $A=[a_{ij}]=\begin{bmatrix}a_{11} & a_{12}\\ a_{21} & a_{22}\end{bmatrix}$. That gives, $A=\begin{bmatrix}\frac{1}{3} & 0\\ 3 & \frac{4}{3}\end{bmatrix}\therefore A'=\begin{bmatrix}\frac{1}{3} & 3\\ 0 & \frac{4}{3}\end{bmatrix}$.

06. If the distance travelled by a particle in t seconds is given by $S=72t+3t^2-t^3$, then time taken by the particle to come to rest is

(a) 4 seconds (b) 6 seconds (c) 3 seconds (d) 0 seconds

Sol. (b) For $S=72t+3t^2-t^3$, $\frac{dS}{dt}=72+6t-3t^2=-3(t^2-2t-24)$.

When particle comes to rest, then $\frac{dS}{dt}=-3(t^2-2t-24)=-3(t-6)(t+4)=0$

$\therefore t=6$ sec $[\because t\neq -4$

12. The sum of the order and the degree of the differential equation $4\left[\frac{d^2y}{dx^2}\right]^2+3\left[1+\left(\frac{dy}{dx}\right)^2-y\right]=0$ is

(a) 2 (b) 3 (c) not defined (d) 4

Sol. (d) Order : 2, Degree : 2.

$\therefore$ Required sum $=2+2=4$.

18. The probability that it will rain tomorrow in cities A, B and C is 60%, 70% and 80% respectively. The probability that it will rain tomorrow in at least one of the cities is

(a) $\frac{3}{250}$ (b) $\frac{244}{250}$ (c) 1 (d) $\frac{9}{10}$

Sol. (b) Let P(A), P(B) and P(C) be the probabilities of rain in cities A, B, and C respectively.

$\therefore P(A)=\frac{60}{100}, P(B)=\frac{70}{100}, P(C)=\frac{80}{100}$ $\Rightarrow P(A')=\frac{40}{100}, P(B')=\frac{30}{100}, P(C')=\frac{20}{100}$

Now P(rains in at least one city) $=1-$ P(no rain in any city)

$=1-P(A')\,P(B')\,P(C')$

$=1-\frac{40}{100}\times\frac{30}{100}\times\frac{20}{100}=1-\frac{2}{5}\times\frac{3}{10}\times\frac{1}{5}=\frac{250-6}{250}=\frac{244}{250}$.

23. Find the interval (s) in which the function $f(x)=\frac{x}{\log x}$, where $x\in(0,1)\cup(1,\infty)$, is increasing.

Sol. Given $f(x)=\frac{x}{\log x}$

$\Rightarrow f'(x)=\frac{\log x\times 1-x\times\frac{1}{x}}{(\log x)^2}=\frac{\log x-1}{(\log x)^2}$

For the function f to be increasing, we must have $f'(x)\geq 0$

$\Rightarrow \dfrac{\log x - 1}{(\log x)^2} \geq 0$

$\Rightarrow \log x - 1 \geq 0$

$\Rightarrow \log x \geq 1$ $[\because \log e = 1$

$\Rightarrow x \geq e$

$\therefore f(x)$ is increasing on $[e, \infty)$.

24. If $(2\hat{i}+3\hat{j}+\hat{k})$ and $(2\hat{i}+\hat{j}-\hat{k})$ represent the sides $\overrightarrow{AB}$ and $\overrightarrow{AC}$ respectively of ΔABC, find the vector representing the median through A.

Sol. Let AD be the median through A. Clearly, D will be the midpoint of BC.

Therefore, $\overrightarrow{AD} = \dfrac{\overrightarrow{AB}+\overrightarrow{AC}}{2} = \dfrac{(2\hat{i}+3\hat{j}+\hat{k})+(2\hat{i}+\hat{j}-\hat{k})}{2}$

$\Rightarrow \overrightarrow{AD} = 2\hat{i}+2\hat{j}$

Hence, the vector representing the median through $A = 2\hat{i}+2\hat{j}$.

27. If $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$, then compute $A^2 - 4A - 5I$.

Sol. Since $A^2 = AA = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix} = \begin{bmatrix} 9 & 8 & 8 \\ 8 & 9 & 8 \\ 8 & 8 & 9 \end{bmatrix}$.

Now $A^2 - 4A - 5I = \begin{bmatrix} 9 & 8 & 8 \\ 8 & 9 & 8 \\ 8 & 8 & 9 \end{bmatrix} - 4\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix} - 5\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\Rightarrow = \begin{bmatrix} 9 & 8 & 8 \\ 8 & 9 & 8 \\ 8 & 8 & 9 \end{bmatrix} - \begin{bmatrix} 4 & 8 & 8 \\ 8 & 4 & 8 \\ 8 & 8 & 4 \end{bmatrix} - \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = O.$

29. Solve the following Linear Programming Problem graphically.

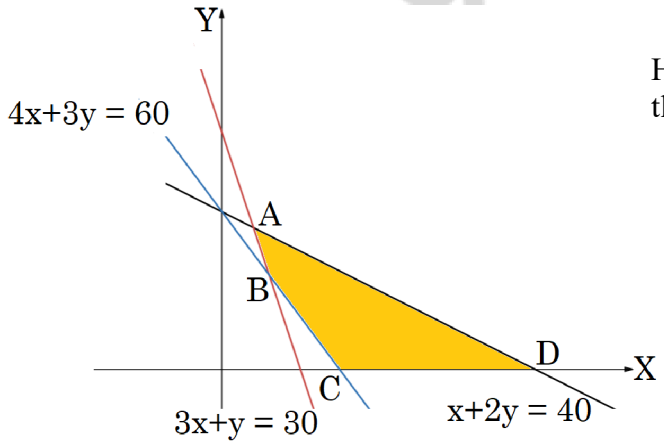
Minimise $Z = 20x + 10y$

Subject to $x + 2y \leq 40$, $3x + y \geq 30$, $4x + 3y \geq 60$; $x, y \geq 0$.

Sol. Consider the graph shown.

The corner points are A(4, 18), B(6, 12), C(15, 0) and D(40, 0).

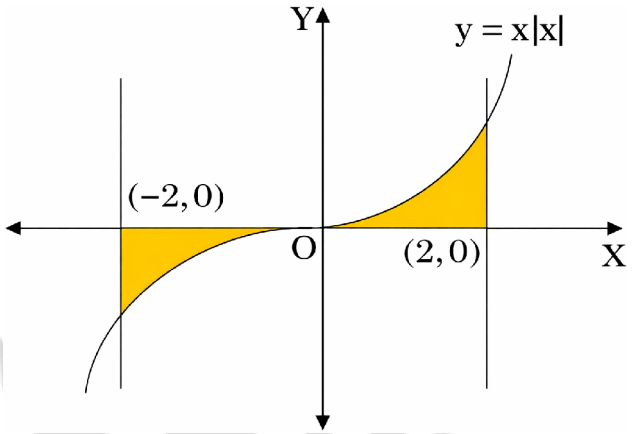
Therefore, $Z_A = 260$, $Z_B = 240$, $Z_C = 300$, $Z_D = 800$.



Hence, the minimum value of Z is 240 at the point (6, 12).

33. Using integration, find the area of the region bounded by the curve $y = x|x|$, x-axis, $x = -2$ and $x = 2$.

Sol. We have the curve $y = x|x| = \begin{cases} x^2, & x \geq 0 \\ -x^2, & x < 0 \end{cases}$.



$$\text{Required area} = \left|\int_{-2}^{0} -x^2 dx\right| + \int_{0}^{2} x^2 dx$$

$$= \left|-\frac{1}{3}\left[x^3\right]_{-2}^{0}\right| + \frac{1}{3}\left[x^3\right]_{0}^{2}$$

$$= \left|-\frac{1}{3}[0-(-8)]\right| + \frac{1}{3}[8-0]$$

$$= \left|-\frac{8}{3}\right| + \frac{8}{3}$$

$$= \frac{8}{3} + \frac{8}{3}$$

$$= \frac{16}{3} \text{ Sq. units}.$$

35. Find the vector and Cartesian equations of the line passing through the point of intersection of the lines $\vec{r} = (\hat{i} + \hat{j} - \hat{k}) + \lambda(3\hat{i} - \hat{j})$ and $\vec{r} = (4\hat{i} - \hat{k}) + \mu(2\hat{i} + 3\hat{k})$ and parallel to the line $\frac{x-1}{-2} = \frac{7-y}{-3} = z$.

Sol. Let $L_1 : \vec{r} = (\hat{i} + \hat{j} - \hat{k}) + \lambda(3\hat{i} - \hat{j})$ and $L_2 : \vec{r} = (4\hat{i} - \hat{k}) + \mu(2\hat{i} + 3\hat{k})$.

The coordinates of any random point on these lines are respectively $P(3\lambda+1, -\lambda+1, -1)$ and $Q(2\mu+4, 0, 3\mu-1)$.

At the point of intersection, the coordinates of P and Q must be equal.

So, $3\lambda+1 = 2\mu+4 \ldots(i)$, $-\lambda+1 = 0 \ldots(ii)$, $-1 = 3\mu-1 \ldots(iii)$.

From (ii) and (iii), we get $\lambda = 1$, $\mu = 0$.

Put values of $\lambda = 1$, $\mu = 0$ in (i).

LHS of (i): $3\lambda+1 = 3\times1+1 = 4$; RHS of (i): $2\mu+4 = 2\times0+4 = 4$.

Clearly, the points P and Q both coincide. So, the lines L_1 and L_2 intersect.

$\therefore$ Point of intersection P(4, 0, –1).

Let $L : \frac{x-1}{-2} = \frac{7-y}{-3} = z$.

To find the d.r.'s of the line L, we shall rewrite it as $L : \frac{x-1}{-2} = \frac{y-7}{3} = \frac{z}{1}$.

Therefore, the d.r.'s of line L are –2, 3, 1.

So, the d.r.'s of required line passing through (4, 0, –1) and parallel to line L are –2, 3, 1.

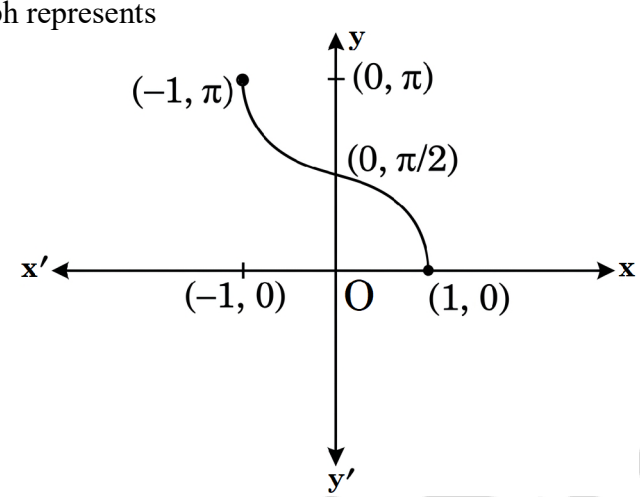
[Since the d.r.'s of parallel lines are proportional

Hence, the required vector equation of line is $\vec{r} = (4\hat{i} - \hat{k}) + \lambda(-2\hat{i} + 3\hat{j} + \hat{k})$.

Also the Cartesian equation of line is $\frac{x-4}{-2} = \frac{y-0}{3} = \frac{z-(-1)}{1}$ i.e., $\frac{x-4}{-2} = \frac{y}{3} = \frac{z+1}{1}$.

☑ Q.P. Code - 65/4/3

01. The following graph represents



(a) $y = \cos^{-1}x$ (b) $y = \sec^{-1}x$ (c) $y = \sin^{-1}x$ (d) $y = \tan^{-1}x$

Sol. (a) The graph is for $y = \cos^{-1}x$. Since domain is $x \in [-1, 1]$ and range is $[0, \pi]$.

02. Let $A = [a_{ij}]$ be a 2×2 matrix whose elements are given by $a_{ij} = \frac{(i+2j)^2}{3}$. Then A' is

(a) $\begin{bmatrix} 12 & \frac{25}{3} \\ \frac{16}{3} & 3 \end{bmatrix}$ (b) $\begin{bmatrix} 12 & \frac{16}{3} \\ \frac{25}{3} & 3 \end{bmatrix}$ (c) $\begin{bmatrix} 3 & \frac{25}{3} \\ \frac{16}{3} & 12 \end{bmatrix}$ (d) $\begin{bmatrix} 3 & \frac{16}{3} \\ \frac{25}{3} & 12 \end{bmatrix}$

Sol. (d) Since for matrix A, $a_{ij} = \frac{(i+2j)^2}{3}$. Then for A', let element $b_{ij} = \left[a_{ij}\right]' = \frac{(j+2i)^2}{3}$.

Let $A' = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} = \begin{bmatrix} 3 & \frac{16}{3} \\ \frac{25}{3} & 12 \end{bmatrix}$.

06. A cylindrical tank is being filled with sand at a rate of 314 m^3/h. If the radius of the tank is 10 m, then the height of sand in the tank increases at the rate of

(a) 1.1 m/h (b) 1 m/h (c) π m/h (d) $\frac{\pi}{2}$ m/h

Sol. (b) Since volume of cylindrical tank is, $V = \pi r^2 h \quad \Rightarrow V = \pi(10)^2 h = 100\pi h$

$\Rightarrow \frac{dV}{dt} = 100\pi \frac{dh}{dt}$

$\Rightarrow 314 = 100 \times 3.14 \times \frac{dh}{dt}$

$\therefore \frac{dh}{dt} = 1$ m/h.

12. The sum of the order and the degree of the differential equation $y = x\frac{dy}{dx} + 3\left[2 - \left(\frac{dy}{dx}\right)^2\right]$ is

(a) not defined (b) 3 (c) 1 (d) 2

Sol. (b) Order : 1, Degree : 2.

$\therefore$ Required sum $= 1 + 2 = 3$.

18. The probability that a particular item is available in three shops A, B and C is $\frac{4}{5}, \frac{3}{4}$ and $\frac{2}{3}$ respectively. If a person visits all the three shops to buy the item, then what is the probability that it will be available in at least one of the shops?

(a) $\frac{59}{60}$ (b) 1 (c) $\frac{1}{60}$ (d) $\frac{9}{60}$

Sol. (a) Required probability $= 1 - P(A')\, P(B')\, P(C')$

$$= 1 - \left[1 - \frac{4}{5}\right]\left[1 - \frac{3}{4}\right]\left[1 - \frac{2}{3}\right] = 1 - \frac{1}{5} \times \frac{1}{4} \times \frac{1}{3} = \frac{60 - 1}{60} = \frac{59}{60}.$$

23. Find the interval (s) for which the function $f(x) = \frac{x}{3} + \frac{3}{x}$, $x \neq 0$ is increasing.

Sol. Given $f(x) = \frac{x}{3} + \frac{3}{x}$, $x \neq 0$

$\Rightarrow f'(x) = \frac{1}{3} - \frac{3}{x^2}$

For critical points, $f'(x) = 0$

$\Rightarrow \frac{1}{3} - \frac{3}{x^2} = 0$

$\Rightarrow x^2 = 9$

$\therefore x = \pm 3$

Note that $f(x)$ is not defined at $x = 0$.

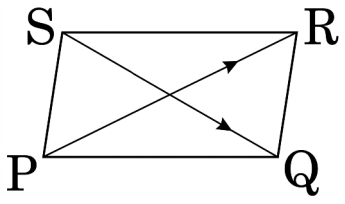
Interval	Sign of $f'(x)$	$f(x)$ is
$(-\infty, -3]$	Positive	Increasing
$[-3, 0)$	Negative	Decreasing
$(0, 3]$	Negative	Decreasing
$[3, \infty)$	Positive	Increasing

$\therefore f(x)$ is increasing on $(-\infty, -3] \cup [3, \infty)$.

24. If in a parallelogram PQRS, $\overrightarrow{PQ} = 2\hat{i} + 3\hat{j} + \hat{k}$ and $\overrightarrow{QR} = \hat{i} - 2\hat{j} + \hat{k}$, then find the unit vectors parallel to the diagonals $\overrightarrow{PR}$ and $\overrightarrow{SQ}$.

Sol. The diagonal $\overrightarrow{PR} = \overrightarrow{PQ} + \overrightarrow{QR} = 3\hat{i} + \hat{j} + 2\hat{k} = \vec{d}_1$.

Also $\overrightarrow{PS} + \overrightarrow{SQ} = \overrightarrow{PQ} \Rightarrow \overrightarrow{SQ} = \overrightarrow{PQ} - \overrightarrow{QR} = \hat{i} + 5\hat{j} = \vec{d}_2$.



Note that, $\overrightarrow{PS} = \overrightarrow{QR}$ in the parallelogram PQRS.

Therefore, $\hat{d}_1 = \frac{\vec{d}_1}{|\vec{d}_1|} = \frac{3\hat{i} + \hat{j} + 2\hat{k}}{\sqrt{9 + 1 + 4}} = \frac{3\hat{i} + \hat{j} + 2\hat{k}}{\sqrt{14}}$.

Also $\hat{d}_2 = \frac{\vec{d}_2}{|\vec{d}_2|} = \frac{\hat{i} + 5\hat{j}}{\sqrt{1 + 25}} = \frac{\hat{i} + 5\hat{j}}{\sqrt{26}}$.

27. If $A = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$, then compute $A^2 - 5A + 4I$.

Sol. $A^2 = AA = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}\begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix} = \begin{bmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{bmatrix}$

So, $A^2 - 5A + 4I = \begin{bmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{bmatrix} - 5\begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix} + 4\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{bmatrix} - \begin{bmatrix} 10 & 0 & 5 \\ 10 & 5 & 15 \\ 5 & -5 & 0 \end{bmatrix} + \begin{bmatrix} 4 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}$

$= \begin{bmatrix} 9 & -1 & 2 \\ 9 & 2 & 5 \\ 0 & -1 & 2 \end{bmatrix} - \begin{bmatrix} 10 & 0 & 5 \\ 10 & 5 & 15 \\ 5 & -5 & 0 \end{bmatrix} = \begin{bmatrix} -1 & -1 & -3 \\ -1 & -3 & -10 \\ -5 & 4 & 2 \end{bmatrix}$

29. Solve the following Linear Programming Problem graphically.

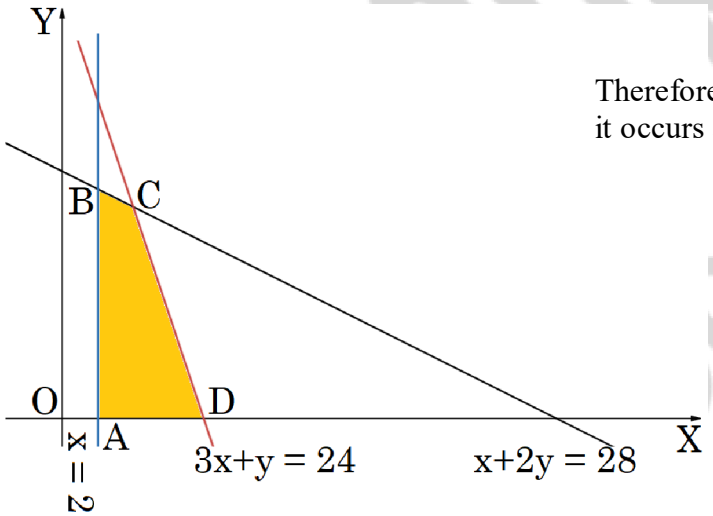
Maximise $Z = 20x + 10y$

Subject to constraints $x + 2y \le 28$, $3x + y \le 24$, $x \ge 2$; $x \ge 0$, $y \ge 0$.

Sol. Consider the graph shown.

The corner points are A(2, 0), B(2, 13), C(4, 12) and D(8, 0).

Therefore, $Z_A = 40$, $Z_B = 170$, $Z_C = 200$, $Z_D = 160$.

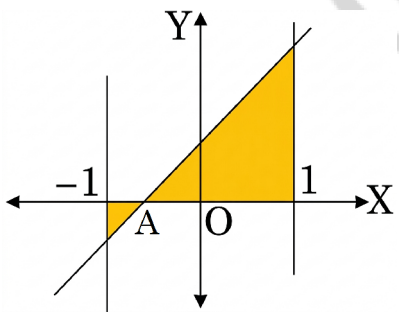


Therefore, the maximum value of Z is 200 and it occurs at the point (4, 12).

33. Using integration, find the area of the region bounded by $y = 5x + 4$, $y = 0$, $x = -1$ and $x = 1$.

Sol. Given $y = 5x + 4$, $y = 0$, $x = -1$ and $x = 1$.

Note that, $y = 5x + 4$ cuts x-axis at $A\left(-\frac{4}{5}, 0\right)$.



$\therefore$ Required area $= \left|\int_{-1}^{-\frac{4}{5}} (5x+4)dx\right| + \int_{-\frac{4}{5}}^{1} (5x+4)dx$

$= \left|\frac{1}{2\times 5}\left[(5x+4)^2\right]_{-1}^{-\frac{4}{5}}\right| + \frac{1}{2\times 5}\left[(5x+4)^2\right]_{-\frac{4}{5}}^{1}$

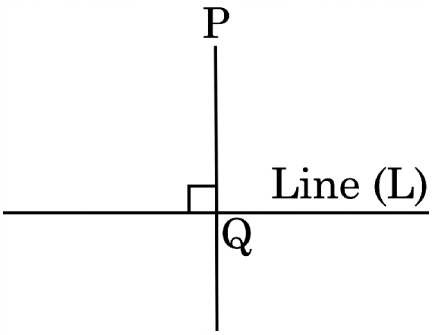
$= \left|\frac{1}{10}[0-1]\right| + \frac{1}{10}[81-0] = \frac{1}{10} + \frac{81}{10} = \frac{82}{10} = \frac{41}{5}$ Sq. units.

35. Find the length of the perpendicular drawn from the point P(1, 2, 3) to the line $\frac{x-6}{3}=\frac{y-7}{2}=\frac{z-7}{-2}$. Also, find the equation of the perpendicular line joining P and the foot of the perpendicular.

Sol. Let $L: \frac{x-6}{3}=\frac{y-7}{2}=\frac{z-7}{-2}=\lambda$.

The d.r.'s of line L are 3, 2, –2.

Draw PQ ⊥ line L.



The coordinates of a random point on line L :

$Q(3\lambda+6,\ 2\lambda+7,\ -2\lambda+7)$.

The d.r.'s of line PQ : $3\lambda+6-1,\ 2\lambda+7-2,\ -2\lambda+7-3$

i.e., $3\lambda+5,\ 2\lambda+5,\ -2\lambda+4$.

Since $a_1a_2+b_1b_2+c_1c_2=0$ for $\perp^{er}$ lines.

So, $3(3\lambda+5)+2(2\lambda+5)+(-2)(-2\lambda+4)=0$

$\Rightarrow 17\lambda+17=0$

$\Rightarrow \lambda=-1$

So, the Foot of perpendicular is $Q(3, 5, 9)$.

Therefore, the length of perpendicular $(PQ)=\sqrt{(3-1)^2+(5-2)^2+(9-3)^2}$

$\Rightarrow (PQ)=\sqrt{4+9+36}=\sqrt{49}=7$ units.

Also equation of line (PQ) : $\frac{x-1}{3-1}=\frac{y-2}{5-2}=\frac{z-3}{9-3}$ i.e., $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{6}$.

With regards,
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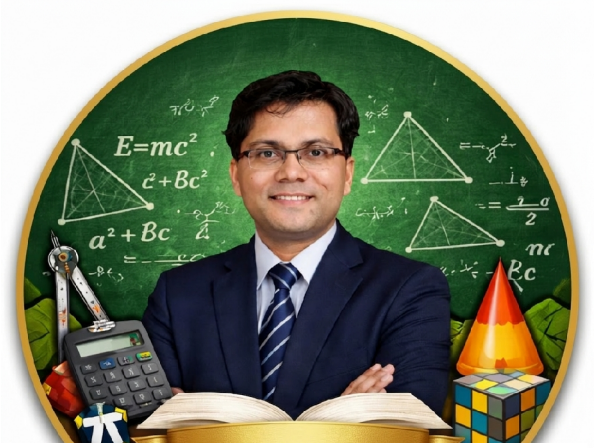
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